

Modelling and Analysis of Stick-slip Behaviour in a Drillstring under Dry Friction*

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Abstract

Stick-slip vibrations present at the bottom hole assembly (BHA) of a generic oilwell drillstring subject to dry friction are studied. Different nonlinear differential equations-based models of a drillstring are analyzed. The drillstring models presented are oriented to avoid simulation problems due to the discontinuities originated by the presence of dry friction. The influence of different drilling parameters on the stick-slip behaviour are studied and relations between them are established to briefly state some control methodologies for drilling operation.

1 Introduction

Mechanical vibrations affecting the bottom-hole assembly (BHA) and the drillstring are a major cause of premature bit and drillstring components failures. Drillstring vibrations are classified depending on the direction they appear, then three main types of vibrations are distinguished, such as: torsional (for example, stick-slip vibrations), axial (for instance, bit bouncing phenomenon) and lateral (for example, whirl motion). This paper is focused on stick-slip vibrations given at the BHA. The associated phenomenon is that the top of the drillstring rotates with a constant rotary speed, whereas the bit rotary speed varies between zero and up to six times the rotary speed measured at surface.

One of the causes of drillstring vibrations (mainly, stick-slip and lateral vibrations) is the friction appearing between the different components of the drillstring and the friction originated from the interaction of the BHA and drillstring components with the formation, see [8]. Consequently, friction will be an important phenomenon

to model and be taken into account in order to analyze the drillstring behaviour. A fair amount of studies concerning self-excited stick-slip oscillations in mechanical systems has dealt with the modelling and the influence of friction in the response of systems, see [2, 5] and references therein. Further analysis has to be done in order to consider the discontinuities at velocities zero derived from the friction presented, this is a key feature in order to apply control methodologies to improve the performance of systems under friction.

One of the main features of friction is that it is not zero at zero velocity, and this is explained by means of the stiction or static friction model, i.e., the external forces applied to an object must overcome a threshold (break-away force) in order to make it start moving along a surface. This force is greater than that needed to keep the object in motion. This phenomenon is usually modelled by a discontinuous classical model of static friction plus Coulomb friction, which is known as dry friction, see [2], and will be used in this paper.

Models describing the drillstring behaviour include the effect of friction appearing between the drillstring components and between the drillstring and the formation. This paper uses lumped parameter differential equations-based models; the drillstring is considered as a torsional pendulum with two degrees of freedom. Most of the models describing stick-slip motion in drillstrings consider the drillstring as a torsional pendulum with different degrees of freedom, for instance: [8, 13, 16] propose single-degree-of-freedom models, [1, 3] propose two-degree-of-freedom models including a linear controller, and [6, 10, 15] present two-degree-of-freedom models for the mechanical part of the system plus the model for the rotary table electric motor system.

Manipulating different drilling parameters as increasing the rotary speed, decreasing the weight on the bit (W_{ob}) or modifying the drilling mud characteristics are shown in the field to suppress stick-slip motion [14].

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Other control methodologies have also appeared in the literature in order to compensate drillstring stick-slip vibrations. These methods can be divided into classical control techniques and more sophisticated ones. In the first group, the following ones are highlighted: (i) introduction of a vibration absorber (regarded as soft torque rotary system) at the top of the drillstring [6] which follows the same approach given in [4, 14]; (ii) introduction of a PID controller at the surface in order to control the rotary speed [12]; (iii) introduction of an additional friction at the bit [12]. A more sophisticated control methodology is used in [15] where a linear H_∞ control is used to suppress stick-slip motion at the bit.

The paper is organized as follows. Section 2 presents the model to describe the torsional behaviour of a simplified conventional drillstring. Alternative models for the dry friction included in the model will be proposed in Section 3. These models will be obtained to have more appropriate friction representations for the system simulation. Analysis of the stick-slip oscillations will be given in Section 4. Conclusions and comments on future works are given in the last section.

2 Modelling the drillstring torsional behaviour

Two main modelling problems have to be distinguished when the drillstring behaviour is treated. On the one hand, the problem of modelling the drilling system, in this case, the torsional behaviour. It is supposed that no lateral motion of the bit is present. On the other hand, the problem of modelling the interaction of the rock (type of well) with different drillstring components (mainly, the rock-bit interaction). This interaction is usually modelled by frictional forces, and in this paper, the rock-bit interaction is simplified by means of a dry friction model.

The model used for describing the torsional behaviour of the drillstring is a simple torsional pendulum driven by an electric motor (see Fig. 1). The equations of motion are the following ones [11]:

$$J_r \ddot{\varphi}_r + c(\dot{\varphi}_r - \dot{\varphi}_b) + k(\varphi_r - \varphi_b) = T_m - T_d(\dot{\varphi}_r) \quad (1a)$$

$$J_b \ddot{\varphi}_b - c(\dot{\varphi}_r - \dot{\varphi}_b) - k(\varphi_r - \varphi_b) = -T_b(\dot{\varphi}_b) \quad (1b)$$

The drill pipes are represented as a torsional spring of stiffness k and the drill collars are considered as rigid bodies. k and a damping coefficient c are connected to the inertias J_r and J_b , corresponding to the inertia of the rotary table and to the inertia of the pipeline plus the down-hole end. J_b is usually considered as the sum of the BHA inertia and one third of the drill pipes inertia, see [3]. The model presented is similar to the ones given in [10] and it is inspired in the ones presented in [6, 15]. Here, the

dynamics of the electric motor system in the rotary table is not considered and the weight on bit is included.

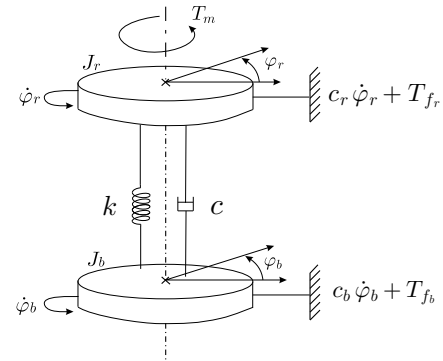


Figure 1: Mechanical model describing the torsional behaviour of a generic drillstring.

φ_r is the angular displacement of the rotary table, φ_b the angular displacement of the bit and the drill collars, T_m the drive torque coming from the rotary table transmission box which is driven by a DC electric motor, and it will be considered as $T_m = k_m u$ with k_m the motor parameter and u the input to the system. T_d and T_b are the friction torques associated with J_r and J_b , respectively. T_b represents the torque-on-bit (TOB) and the nonlinear frictional forces along the drill collars. Torques T_d and T_b have the following form,

$$T_d(\dot{\varphi}_r) = c_r \dot{\varphi}_r + T_{f_r}(\dot{\varphi}_r) \quad (2a)$$

$$T_b(\dot{\varphi}_b) = c_b \dot{\varphi}_b + T_{f_b}(\dot{\varphi}_b) \quad (2b)$$

with c_r and c_b the damping viscous coefficients associated with the rotary table and the bit, respectively. The expression for either T_{f_r} or T_{f_b} is the classical Coulomb plus static friction (dry friction) model [2], that is,

$$T_{f_i}(\dot{\varphi}_i) = \begin{cases} T_{c_i} \text{sign}(\dot{\varphi}_i) & \text{if } \dot{\varphi}_i \neq 0 \\ |T_{f_i}| \leq T_{s_i} & \text{if } \dot{\varphi}_i = 0 \end{cases} \quad (3)$$

with $i \in \{r, b\}$, $T_{s_r} > 0$ and $0 < T_{c_r} < T_{s_r}$ are the static and Coulomb friction torques, respectively, associated with inertia J_r ; $T_{s_b} = \mu_{s_b} W_{ob} R_b$ and $T_{c_b} = \mu_{c_b} W_{ob} R_b$ the static and Coulomb friction torques associated with inertia J_b ; $\mu_{s_b}, \mu_{c_b} \in (0, 1)$, the static and Coulomb friction coefficients associated with inertia J_b , $\mu_{s_b} > \mu_{c_b}$; $W_{ob} > 0$ is the weight on the bit and $R_b > 0$ is the bit radius.

Let us define the state vector $x = (\varphi_r, \dot{\varphi}_r, \varphi_b, \dot{\varphi}_b)^T$.

The state-space representation of system (1) is,

$$\dot{x} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_r} & -\frac{c+c_r}{J_r} & \frac{k}{J_r} & \frac{c}{J_r} \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_b} & \frac{c}{J_b} & -\frac{k}{J_b} & -\frac{c+c_b}{J_b} \end{pmatrix} x + \begin{pmatrix} 0 \\ \frac{k_m u - T_{fr}(\dot{\phi}_r)}{J_r} \\ 0 \\ -\frac{T_{fb}(\dot{\phi}_b)}{J_b} \end{pmatrix} \quad (4)$$

The critical points of (4) are given by the following set:

$$\mathcal{X} = \left\{ x : \dot{\phi}_r = \dot{\phi}_b = 0, \frac{T_m - T_{sr}}{k} \leq \phi_r - \phi_b \leq \frac{T_{sr} + T_m}{k} \right\}$$

with T_m such that $|u| \leq \frac{T_{sr} + T_{sb}}{k_m}$.

Model (4) with (3) will not be used for analysis purposes in this paper, it will be rewritten in different ways in Sections 3 and 4. In Section 3, different models for the dry friction appearing at the rotary table and the BHA will be given. These models are oriented to achieve an adequate representation of the model for simulation purposes. On the other hand, in Section 4, a simplified model of (4) will be considered only for the case of velocities $\dot{\phi}_r$, $\dot{\phi}_b$ being greater than zero.

3 Alternative models for the friction appearing in the drillstring

Special attention must be paid to mechanical systems under dry friction described by a discontinuous model. The main problem is the multivalued character of the friction torque at velocity zero, consequently, the function describing the friction must be interpreted appropriately.

This section presents three alternative models for the friction at the rotary table and the one between the bit and the rock with the form (3). In Section 4.1, the friction models presented will be used in order to simulate model (4) and study some aspects of drillstring stick-slip vibrations.

3.1 Combination of dry friction, switch and Karnopp's models

The first alternative to friction model (3) is a combination of the switch model proposed in [9] and the dry friction model (3) in which a zero velocity band is introduced,

i.e., Karnopp's model [7], see Fig. 2.1. Then,

$$T_{fi}(x) = \begin{cases} T_{ei}(x) & \text{if } |\dot{\phi}_i| < D_v, |T_{ei}| \leq T_{si} \\ & \text{(stick)} \\ T_{si} \text{sign}(T_{ei}) & \text{if } |\dot{\phi}_i| < D_v, |T_{ei}| > T_{si} \\ & \text{(stick-to-slip transition)} \\ T_{ci} \text{sign}(\dot{\phi}_i) & \text{if } |\dot{\phi}_i| \geq D_v \\ & \text{(slip)} \end{cases} \quad (5)$$

where $i \in \{r, b\}$, x is the system state vector, $D_v > 0$ specifies a small enough neighbourhood of $\dot{\phi}_i = 0$, $T_{er}(x)$ and $T_{eb}(x)$ are the applied external torques that must overcome the static friction torques T_{sr} and T_{sb} to make the rotary table and the bit move, respectively, and they have the following form:

$$\begin{aligned} T_{er}(x) &= T_m - c(\dot{\phi}_r - \dot{\phi}_b) - k(\phi_r - \phi_b) - c_r \dot{\phi}_r \\ T_{eb}(x) &= c(\dot{\phi}_r - \dot{\phi}_b) + k(\phi_r - \phi_b) - c_b \dot{\phi}_b \end{aligned} \quad (6)$$

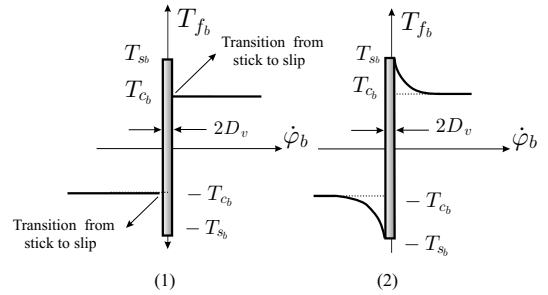


Figure 2: Models for the friction between the bit and the rock: (1) T_{fb} given by expression (5); (2) T_{fb} given by expressions (7)-(8), (7)-(9).

3.2 Karnopp's friction model with a decaying quotient -type friction in the slipping phase

In this section, use is made of Karnopp's friction model. The function governing friction in the slipping phase is chosen as a decaying function inspired in the experimental results given in [3, 12] and the proposal of [9], see Fig. 2.2. The result is:

$$T_{fi}(x) = \begin{cases} \min\{|T_{ei}|, T_{si}\} \text{sgn}(T_{ei}) & \text{if } |\dot{\phi}_i| < D_v \\ f_i(\dot{\phi}_i) \text{sign}(\dot{\phi}_i) & \text{if } |\dot{\phi}_i| \geq D_v \end{cases} \quad (7)$$

with $i \in \{r, b\}$, and T_{er} , T_{eb} are defined in (6). Functions f_r and f_b have the following form:

$$\begin{aligned} f_r(\dot{\phi}_r) &= \frac{T_{sr} - T_{cr}}{1 + \gamma_r |\dot{\phi}_r|} + T_{cr} \\ f_b(\dot{\phi}_b) &= W_{ob} R_b \mu_b(\dot{\phi}_b) \\ \mu_b(\dot{\phi}_b) &= \frac{\mu_{sb} - \mu_{cb}}{1 + \gamma_b |\dot{\phi}_b|} + \mu_{cb} \end{aligned} \quad (8)$$

with μ_b the dry friction coefficient at the bit and γ_r, γ_b positive constants defining the decaying velocity of T_{fr} and T_{fb} , respectively. The main difference between (5) and (7) is that in the case of considering (7), four different state cases of system (4) are obtained; whereas (5) leads up nine state cases.

3.3 Karnopp's friction model with a decaying exponential-type friction in the slipping phase

The third proposal to model the dry-type frictions appeared in the drillstring description (4) is Karnopp's model with an exponential-type function describing the friction in the slipping phase. This kind of friction at non-zero angular velocity is inspired in the one proposed in [1] and have the expression given in (7) with:

$$\begin{aligned} f_r(\dot{\phi}_r) &= T_{cr} + (T_{sr} - T_{cr})e^{-\gamma_r|\dot{\phi}_r|} \\ f_b(\dot{\phi}_b) &= W_{ob}R_b\mu_b(\dot{\phi}_b) \\ \mu_b(\dot{\phi}_b) &= \mu_{cb} + (\mu_{sb} - \mu_{cb})e^{-\gamma_b|\dot{\phi}_b|} \end{aligned} \quad (9)$$

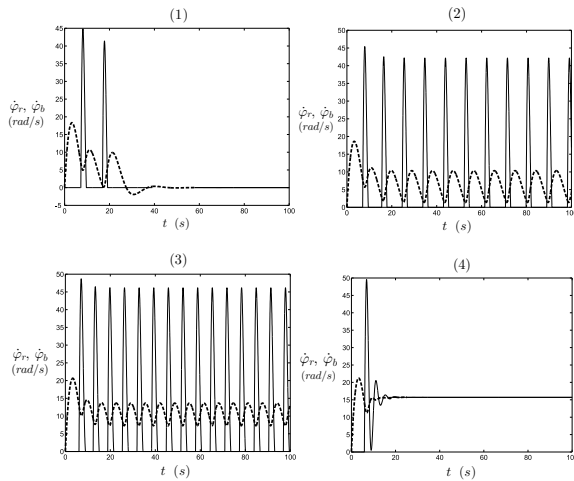


Figure 3: Angular velocities at the rotary table (---) and the BHA (—) using (5) for different values of u , with $c_b = 0.03 \text{ Nms/rad}$, $W_{ob} = 100 \text{ N}$: (1) $u = 7.3 \text{ Nm}$; (2) stick-slip at the bit with $u = 7.4 \text{ Nm}$; (3) $u = 8.1 \text{ Nm}$; (4) suppression of stick-slip with $u = 8.3 \text{ Nm}$.

4 Analysis of the stick-slip model

The purpose of this section is twofold. First, using the drillstring model presented in previous sections, the influence of some drilling parameters in the stick-slip behaviour will be discussed. The conclusions achieved are in accordance with field experimental results. Second, a reduced-order drillstring model will be used in order to

study the behaviour of the system in the slipping phase, when no stick is present.

4.1 Influence of model parameter in stick-slip oscillations

Model (4) appropriately describes the stick-slip phenomenon in a generic drillstring. It can be used together with frictions (5)-(9) in order to validate the typical recommendations for suppressing stick-slip oscillations in a drillstring by manipulating some drilling parameters [3].

The main parameters observed to influence the suppression or appearance of stick-slip behaviour are described as follows:

- (i) **Torque supplied by the motor at the rotary table.** There are ranges of u for which stick-slip oscillations are not present, see Fig. 3.
- (ii) **Damping viscous coefficient at the BHA (c_b).** Values of c_b higher than a certain threshold assure the suppression of stick-slip oscillations, see Fig. 4.
- (iii) **Static and "sliding" friction torques $T_{sr}, T_{cb}, T_{sr}, T_{cr}$.** The smaller the differences $T_{sr} - T_{cr}, T_{sb} - T_{cb}$ are, the smaller the possibility of occurrence of stick-slip oscillations at the BHA is, see Fig. 4.
- (iv) **Weight on bit.** For some conditions, the decrease of the W_{ob} suppresses stick-slip oscillations at the bit, see Fig. 4.
- (v) **Rotary angular bit velocity.** Stick-slip motion is not present for rotary velocities higher than a critical value. Notice from Figures 3, 4 that when stick-slip does not occur the rotary velocities are higher than in the stick-slip situation.

The model parameters used for the simulations are presented as follows and have been extracted from [10]:

$$\begin{aligned} J_r &= 0.518 \text{ kg m}^2, J_b = 0.0318 \text{ kg m}^2, \\ c &= 0.0001 \text{ Nms/rad}, k = 0.073 \text{ Nm/rad}, \\ c_r &= 0.18 \text{ Nms/rad}, k_m = 1, \\ \mu_{cb} &= 0.5, \mu_{sb} = 0.8, R_b = 0.1 \text{ m}, D_v = 10^{-6} \end{aligned} \quad (10)$$

Remark 1 Although values (10) do not correspond with real parameters, they can be used to describe the behaviour of the drillstring. The parameters depend on the formation drilled and the type of bit and drilling used.

4.2 Study of the slipping phase

A lower dimension model can be obtained from model (4)-(5) by defining the state $\varphi = \phi_r - \phi_b$, then the state

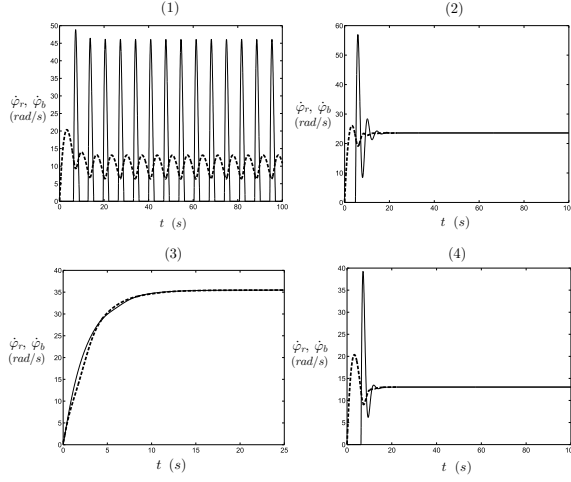


Figure 4: Angular velocities at the rotary table (---) and the BHA (—) with (7)-(9) for different c_b , W_{ob} , T_{sr} , T_{cr} with $\gamma_r = \gamma_b = 0.9$ and $u = 10 \text{ Nm}$. (1) $T_{sr} = 4 \text{ Nm}$, $T_{cr} = 2 \text{ Nm}$, $c_b = 0.03 \text{ Nms/rad}$, $W_{ob} = 100 \text{ N}$; (2) $T_{sr} = 0.3 \text{ Nm}$, $T_{cr} = 0.05 \text{ Nm}$, $c_b = 0.03 \text{ Nms/rad}$, $W_{ob} = 100 \text{ N}$; (3) $T_{sr} = 4 \text{ Nm}$, $T_{cr} = 2 \text{ Nm}$, $c_b = 0.03 \text{ Nms/rad}$, $W_{ob} = 11 \text{ N}$; (4) $T_{sr} = 4 \text{ Nm}$, $T_{cr} = 2 \text{ Nm}$, $c_b = 0.05 \text{ Nms/rad}$, $W_{ob} = 100 \text{ N}$.

vector of the new system is $x = (\dot{\phi}_r, \phi, \dot{\phi}_b)^T$. In this case, the bit is supposed not to rotate backwards, i.e., $\dot{\phi}_b > 0$ and $\dot{\phi}_r > 0$. Consequently, the slipping phase for both the rotary table and the BHA is considered. The equation of motion of the simplified model is the following one:

$$\dot{x} = \begin{pmatrix} -\frac{c+c_r}{J_r} & -\frac{k}{J_r} & \frac{c}{J_r} \\ 1 & 0 & -1 \\ \frac{c}{J_b} & \frac{k}{J_b} & -\frac{c+c_b}{J_b} \end{pmatrix} x + \begin{pmatrix} \frac{k_m u - T_{cr}}{J_r} \\ 0 \\ -\frac{T_{cb}}{J_b} \end{pmatrix} \quad (11)$$

The critical points of (11) are

$$\begin{aligned} \bar{\phi}_r = \bar{\phi}_b = \bar{\Omega} &= -\frac{1}{c_b + c_r} (T_{cb} + T_{cr} - k_m \bar{u}) \\ \bar{\phi} &= -\frac{1}{k(c_b + c_r)} (c_r T_{cb} - c_b T_{cr} + c_b k_m \bar{u}) \end{aligned} \quad (12)$$

Taking into account that $\bar{\Omega} > 0$, from (12), it is obtained that

$$\bar{u} > \frac{T_{cb} + T_{cr}}{k_m} = \bar{u}^* \quad (13)$$

Using (12) and (13), the limits for $\bar{\Omega}$ and $\bar{\phi}$ are obtained,

$$\bar{\Omega} > 0, \quad \bar{\phi} > \frac{T_{cb}}{k} = \bar{\phi}^*$$

Considering $\bar{u} = \delta \bar{u}^*$ with $\delta > 1$, (12) yields to

$$\begin{aligned} \bar{\Omega} &= \frac{(\delta - 1)(T_{cb} + T_{cr})}{c_b + c_r} \\ \bar{\phi} &= \frac{1}{k(c_b + c_r)} [T_{cb}(c_r + c_b \delta) + T_{cr} c_b (\delta - 1)] \end{aligned} \quad (14)$$

The Routh-Hurwitz criteria applied on system (11) assures that all the poles of the system have negative real parts, in conclusion, equilibria (14) for each δ are asymptotically stable.

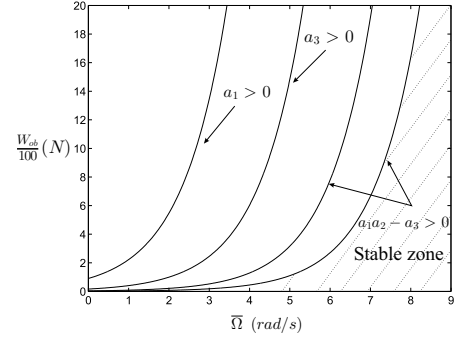


Figure 5: $W_{ob}/\bar{\Omega}$ stability diagram for model (11) with “slipping” frictions (15).

Relations between the stability of the system and some drilling parameters can also be obtained by using model (11) and the exponential-type friction for the slipping phase given in (9). In this case, frictions associated with inertias J_r and J_b take the following form:

$$\begin{aligned} T_{fr}(\dot{\phi}_r) &= T_{cr} + (T_{sr} - T_{cr})e^{-\gamma_r \dot{\phi}_r} \\ T_{fb}(\dot{\phi}_b) &= W_{ob} R_b [\mu_{cb} + (\mu_{sb} - \mu_{cb})e^{-\gamma_b \dot{\phi}_b}] \end{aligned} \quad (15)$$

Consider system (11) with T_{fr} and T_{fb} given in (15). If this system is linearized around $(\bar{\Omega}, \bar{\phi}, \bar{\Omega})^T$, the stability of the system in a neighbourhood of $(\bar{\Omega}, \bar{\phi}, \bar{\Omega})^T$ can be analyzed by means of the eigenvalues of the following matrix:

$$J = \begin{pmatrix} -\frac{c+c_r}{J_r} & -\frac{k}{J_r} & \frac{c}{J_r} - \frac{d_r}{J_r} \\ 1 & 0 & -1 \\ \frac{c}{J_b} & \frac{k}{J_b} & -\frac{c+c_b}{J_b} - \frac{d_b}{J_b} \end{pmatrix} \quad (16)$$

with

$$d_r = \left. \frac{\partial T_{fr}(\dot{\phi}_r)}{\partial \dot{\phi}_r} \right|_{\dot{\phi}_r = \bar{\Omega}}, \quad d_b = \left. \frac{\partial T_{fb}(\dot{\phi}_b)}{\partial \dot{\phi}_b} \right|_{\dot{\phi}_b = \bar{\Omega}}$$

By the Routh-Hurwitz criteria, the conditions for matrix (16) to have eigenvalues with negative real parts are given as conditions over the rotary speed $\bar{\Omega}$ and W_{ob} , that is,

$$a_1 > 0, \quad a_3 > 0, \quad a_1 a_2 - a_3 > 0 \quad (17)$$

where,

$$\begin{aligned} a_1 &= \frac{1}{J_r J_b} [J_b(c + c_r) + J_r(c + c_b + d_b)] \\ a_2 &= \frac{1}{J_r J_b} [c_r(c_b + d_b) + c(c_b + c_r + d_r + d_b) + k(J_b + J_r)] \\ a_3 &= \frac{k(c_b + c_r + d_r + d_b)}{J_r J_b} \end{aligned}$$

Conditions (17) are depicted in Fig. 5. They are similar to those given in [1]. From the stability relations given in Fig. 5, it can be noticed that the reduction of the W_{ob} and the increase of the rotary speed yields stable motions with no stick-slip, which is in accordance with field experiments [3].

Remark 2 Results in Fig. 5 have been obtained using more realistic values for the model parameters, they have been extracted from [15]: $J_r = 2122 \text{ kg m}^2$, $J_b = 374 \text{ kg m}^2$, $k = 473 \text{ Nm/rad}$, $c = 100 \text{ Nms/rad}$, $c_r = 425 \text{ Nms/rad}$, $c_b = 20 \text{ Nms/rad}$, $T_{c_r} = 150 \text{ Nm}$, $T_{s_r} = 400 \text{ Nm}$, $k_m = 1$, $\gamma_r = \gamma_b = 0.9$.

5 Conclusions

This paper has studied stick-slip vibrations in a generic oilwell drillstring. Different models for describing the torsional drillstring behaviour have been given. In these models, the influence of some drilling parameters in the suppression of bit stick-slip oscillations have been evaluated. The conclusions achieved are in accordance with field experiments.

Modelling of oilwell drillstrings oriented to the description of mechanical vibrations and the control of these vibrations are open research problems. In order to have more realistic models, the consideration of drillstring lateral dynamics and the influence of mud drilling fluids are needed. It is also necessary to make an analysis of the influence of the drillstring length, formation properties and bit type in the model parameters.

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